

**Solve.**

- 1) A ball is dropped from a height of 3.0 m. On each upward bounce the ball returns to  $\frac{2}{3}$  of its previous height.

Find the total vertical distance the ball travels before coming to rest.

$$S_{\infty} = \frac{a_1}{1-r} = \frac{3}{1-\frac{2}{3}} = \frac{3}{\frac{1}{3}} = 9 \text{ cm}$$



- 2) Simplify the factorial expression.

$$\frac{(5n+4)!}{(5n+9)!} = \frac{(5n+4)!}{(5n+9)(5n+8)(5n+7)(5n+6)(5n+5)(5n+4)(5n+3)(5n+2)(5n+1)!}$$

Write the sum using summation notation, assuming the suggested pattern continues.

3)  $1 + 4 + 9 + 16 + 25 + \dots$

A)  $\sum_{n=0}^{\infty} n^2$

B)  $\sum_{n=1}^{\infty} (n+1)^2$

C)  $\sum_{n=1}^{\infty} n^2$

D)  $\sum_{n=0}^{\infty} (n-1)^2$

4)  $-8 - 7 - 6 - 5 + \dots + 7$

$a_n = -8 + 1(n-1)$

A)  $\sum_{n=0}^{\infty} -8n$

B)  $\sum_{n=0}^{15} -8n$

C)  $\sum_{n=0}^{15} (-8 + 1n)$

D)  ~~$\sum_{n=0}^{\infty} (-8 + 1n)$~~

5)  $5 - 25 + 125 - 625 + \dots$

$5(-5)^{n-1}$

A)  $\sum_{n=0}^{\infty} 5 \cancel{-5^n}$

B)  $\sum_{n=0}^{\infty} 5(-5)^n$

C)  $\sum_{n=0}^{\infty} 5 \cdot 5^n$

D)  ~~$\sum_{n=0}^{\infty} 5(-5)^{n+1}$~~

6) Use sigma notation to write the sum

$$\frac{3}{2} \cdot \frac{6}{4} + \frac{12}{8} - \frac{24}{16} + \frac{48}{32} - \dots$$

$$\frac{3(2)^{n-1}}{2^n}$$

$$\sum_{n=1}^{\infty} \frac{3(2)^{n-1}}{2^n}$$

Determine whether the infinite geometric series converges. If the series converges, determine the sum.

- 7)  $3 + 6 + 12 + 24 + \dots$

$$\frac{6}{3} = 2$$

$$r = 2$$

$$\frac{12}{6} = 2$$

Diverges

Converges  $-1 < r < 1$

8)  $60 - 12 + \frac{12}{5} - \frac{12}{25} + \dots$

$$-\frac{12}{60} = -\frac{1}{5}$$

$$\frac{\frac{12}{5}}{-\frac{12}{60}} = -\frac{1}{5}$$

$$\begin{aligned} S_{\infty} &= \frac{a_1}{1-r} \\ &= \frac{60}{1 - \left(-\frac{1}{5}\right)} \\ &= \frac{60}{\frac{6}{5}} = 50 \end{aligned}$$

Express the rational number as a fraction of integers. Show all your work.

9)  $0.666\dots$

$$\frac{6}{10} + \frac{6}{100} + \frac{6}{1000}$$

$$a_1 = \frac{6}{10}$$

$$r = \frac{1}{10}$$

$$\begin{aligned} S_{\infty} &= \frac{\frac{6}{10}}{1 - \frac{1}{10}} \\ &= \frac{\frac{6}{10}}{\frac{9}{10}} = \frac{6}{9} = \frac{2}{3} \end{aligned}$$

Find an explicit rule for the nth term of the geometric sequence.

10)  $-1, -3, -9, -27, \dots$

$$a_n = a_1 r^{n-1}$$

$$a_1 = -1 \quad r = 3$$

$$a_n = -1 (3)^{n-1}$$

Using the recursive rule, find the first six terms of the sequence.

11)  $a_1 = 7, a_n = a_{n-1} + 5$

$$a_1 = 7$$

$$a_2 = 7 + 5 = 12$$

$$a_3 = 12 + 5 = 17$$

$$a_4 = 17 + 5 = 22$$

12)  $a_1 = 5$

$$a_2 = 5(2) = 10$$

$$a_3 = 10(2) = 20$$

$$a_4 = 20(2) = 40$$

$$a_5 = 40(2) = 80$$

$$a_6 = 80(2) = 160$$

$$2^{n+1}$$

$$2^{n-1}$$

13) Write the explicit formula for the following sequence  $a_5 = 22 + 5 = 27$

$$4^{n+1} \quad \frac{2}{5}, \frac{4}{9}, \frac{6}{13}, \frac{8}{17}, \dots$$

$$a_6 = 27 + 5 = 32$$

$$a_n = \frac{2^n}{4^{n+1}}$$

Write out the first five terms of the sequence. Assume  $n = 1$

14)  $a_n = n + 7$

$$a_1 = 1 + 7 = 8$$

$$a_2 = 2 + 7 = 9$$

$$a_3 = 3 + 7 = 10$$

$$a_4 = 4 + 7 = 11$$

$$a_5 = 5 + 7 = 12$$

$$a_1 = \frac{2(1)+1}{3(1)} = \frac{3}{3} = 1$$

$$a_4 = \frac{2(4)+1}{3(4)} = \frac{9}{12} = \frac{3}{4}$$

$$15) c_n = \frac{2n+1}{3n}$$

$$a_2 = \frac{2(2)+1}{3(2)} = \frac{5}{6}$$

$$a_5 = \frac{2(5)+1}{3(5)} = \frac{11}{15}$$

$$a_3 = \frac{2(3)+1}{3(3)} = \frac{7}{9}$$

Find an explicit rule for the nth term of the arithmetic sequence.

$$\begin{aligned} 16) 19, 27, 35, 43, \dots \\ 27 - 19 = 8 \\ 35 - 27 = 8 \\ d = 8 \quad a_1 = 19 \\ a_n = 19 + 8(n-1) \quad \text{or} \quad a_n = 19 + 8n - 8 \\ & \qquad \qquad \qquad 8n + 11 \end{aligned}$$

17) Write the first 5 terms of the sequence. Assume n=1

$$a_n = \frac{5^n}{(2n-1)!}$$

$$a_1 = \frac{5(1)}{(2(1)-1)!} = \frac{5}{1!} = 5$$

$$a_2 = \frac{5(2)}{(2(2)-1)!} = \frac{10}{3!} = \frac{10}{6} = \frac{5}{3}$$

$$a_3 = \frac{5(3)}{(2(3)-1)!} = \frac{15}{5!} = \frac{15}{120} = \frac{1}{8}$$

$$a_4 = \frac{5(4)}{(2(4)-1)!} = \frac{20}{7!} = \frac{20}{5040} = \frac{1}{252}$$

$$a_5 = \frac{5(5)}{(2(5)-1)!} = \frac{25}{9!} = \frac{25}{362880} = \frac{5}{65376}$$

Find the sum of the arithmetic sequence.

$$18) 40, 42, 44, 46, \dots, 62$$

$$a_1 = 40$$

$$\begin{aligned} S_{12} &= \frac{12}{2}(40+62) \\ &= 6(102) \\ &= 612 \end{aligned}$$

$$a_2 = 40 + 2(n-1)$$

$$22 = 2n - 2$$

$$24 = 2n$$

$$n = 12$$

Solve.

- 19) An auditorium has 25 rows with 10 seats in the first row, 12 in the second row, 14 in the third row, and so forth.  
How many seats are in the auditorium?

$$a_1 = 10 \quad a_{25} = 58$$

$$n = 25$$

$$S = \frac{25}{2}(10 + 58) = 850$$

$$a_{25} = 10 + 2(25-1)$$

$$10 + 2(24)$$

$$58$$

Find an explicit rule for the nth term of the geometric sequence.

- 20) The second and fifth terms of a geometric sequence are -36 and 2304, respectively.

$$a_2 = -36$$

$$\frac{2304}{-36} = \sqrt{-64} = \sqrt[3]{-3}$$

$$a_5 = 2304$$

$$r = -4$$

$$a_1 = 9$$

$$a_n = 9(-4)^{n-1}$$

Find the sum of the first n terms of the sequence.

$$21) 24, 28, 32, 36, \dots; n = 9$$

$$a_n = 24 + 4(n-1)$$

$$a_9 = 54$$

$$S_9 = \frac{9}{2}(24 + 54)$$

$$= 360$$

$$22) 7, -1, -9, -17, \dots; n = 9$$

$$a_n = 7 - 8(n-1)$$

$$a_9 = -57$$

$$S_9 = \frac{9}{2}(7 - 57)$$

$$= -225$$

$$23) 10, 12, 14, 16, \dots; n = 8$$

$$a_n = 10 + 2(n-1)$$

$$a_9 = 24$$

$$S_8 = \frac{8}{2}(10 + 24)$$

$$= 136$$

Find an explicit rule for the nth term of the arithmetic sequence.

$$24) a_{17} = -97, a_{19} = -283$$

$$d = \frac{-283 - (-97)}{19 - 17} = -93$$

$$a_n = a_1 - 93(n-1)$$

$$-97 = a_1 - 93(17-1)$$

$$a_n = 1391 - 93(n-1)$$

Find the sum of the partial geometric series.

$$25) 3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16}$$

$$n = 5$$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$a_1 = 1391$$

$$S_5 = \frac{3(1 - (\frac{1}{2})^5)}{1 - \frac{1}{2}} = \frac{93}{16}$$

$$26) 1 + -3 + 9 + -27 + 81$$

$$n = 5$$

$$a_1 = 1$$

$$r = -3$$

$$S_5 = \frac{1(1 - (-3)^5)}{1 - (-3)} = \frac{244}{4} = 61$$